INCREASING THE COMPREHENSION OF FUNCTION NOTION FROM VARIABILITY AND DEPENDENCE EXPERIENCED WITHIN CABRI-II

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Abstract.. In a research carried out (Hoyos, Capponi, Geneves, 1998) during the 97-98 school year, we set up a series of learning scenarios in a French Licée, using Cabri-II. A teaching experiment was therein performed among Séconde (15-16 year-old) students, around the task of algebraic inequation problem solving. In the present report, we show how Cabri-II was a significant tool to achieve connections between the algebraic and the graphic representations of the functions. Besides, the analysis of the pupils' interplay within the software indicated that perhaps the connections that students established between the different types of function representations were due, to the experience promoted when they worked within Cabri-II, because the kind of support involved by the prototypes they performed: we are speaking of experiencing variability and dependence, which are both structural aspects to the construction of the function object (Freudenthal, 1983), or which are a part of the object perspective (Sfard, 1991; Kieran, 1992) of the concept of function.

Introduction

Several authors have researched and elaborated around an understanding structure of a complex mathematical domain or contents, in terms of establishing connections among various perspectives or approaches that might be appropriate to attack the mathematical contents in question (Moschkovich, Schoenfeld & Arcavi, 1993), and/or the establishment of connections among different settings, notation systems, and meanings (Confrey, 1993; Noss & Hoyles, 1996) contextualizing such contents.

In this paper, we are trying to show that our experimentation is saying something about the understanding of the notion of function in this sense, for we have witnessed that students in fact establish connections between the graphic and the algebraic representations that come into play through the demand of the obtention of the equation of a curve, and by the resolution of inequations using Cabri II.

Our teaching experiment was carried out with Séconde students (15-16 year-olds), around the task of algebraic inequation problem solving, which is one of the classical topics in the basic algebraic curriculum.

We present here some of our results concerning the accomplishments of the pupils when they were working in the settings we have elaborated using Cabri-II, and the analysis of the students' interaction within the software while they were working the proposed algebraic tasks.

Theoretical Underpinnings

- Cabri-II as a support for the promotion of knowledge: Bellmain (1992), and Noss & Hoyles (1996)

We chose Cabri-II microworld because some characteristics of this microworld incite pupils to the perception of geometrical invariant properties: "nous faisons l'hypothèse que l'ordinateur peut guider l'élève dans l'exploration de dessins en l'incitant à repérer des invariants entre plusieurs dessins, invariants pouvant déboucher sur la mise en évidence de notions et propriétés géométriques générales. En effet, ... il peut proposer plusieurs dessins répresentant un même ensemble de données théoriques. " (See Bellemain 1992).

In fact, the computer is conceived (Noss & Hoyles, 1996) as integrating an educational support system wherein knowledge, the student(s), the teacher, and the learning environment participate. The idea of construction of a meaning under this system makes appeal to the presence of a structure on which the students might construct and re-construct, by means of a (real and virtual) support in such ways as they choose as appropriate to their effort in discerning the meaning of the mathematics in question.

- School Algebraic Tasks in Computerized Environments: Kieran (1996)

In according with Kieran (1992, 1996), we recognize the importance of the role in the curriculum of traditional algebraic tasks, and even when the contents of school algebra is now changing: *" it is time, in my view, for the pendulum to swing back to a more middle ground -but not a return to curricula of the past that were exclusively oriented toward symbol manipulation; nor do we have to do as much of the manipulation as we used to do… Without falling back into the traps of the past, we must continue to make significant room for the literal-symbolic in the content of school algebra."* Kieran (1996) has showed how a computational environment can be used as a basis to approach algebraic symbolism and its transformations.

- Connections between different perspectives and representations as aspects of understanding: (Moschkovich, J., Schoenfeld, A., and Arcavi, A., 1993)
The ways of the experts and/or students attack the tasks on some complex mathematical domain have been the mater to elaborate a profitable approach for both curriculum development and (student and curriculum) assessment:*: "(On) the framework for understanding functions… we pointed to two ways of viewing functions (the process and object perspective) and the three most prominent representations of functions (in tabular, graphical, and algebraic form)" (pp.97).*

The Teaching Experiment

Our teaching experiment was carried out during the 1997-1998 school year in a 10th grade class at a French Licée, and it was based on the instrumentation of several settings of the notion of function, using Cabri II software. Such settings interrelated:
(a)the algebraic representation with a geometrical configuration of the functions; and
(b) the algebraic representation with a graphical configuration of such functions.

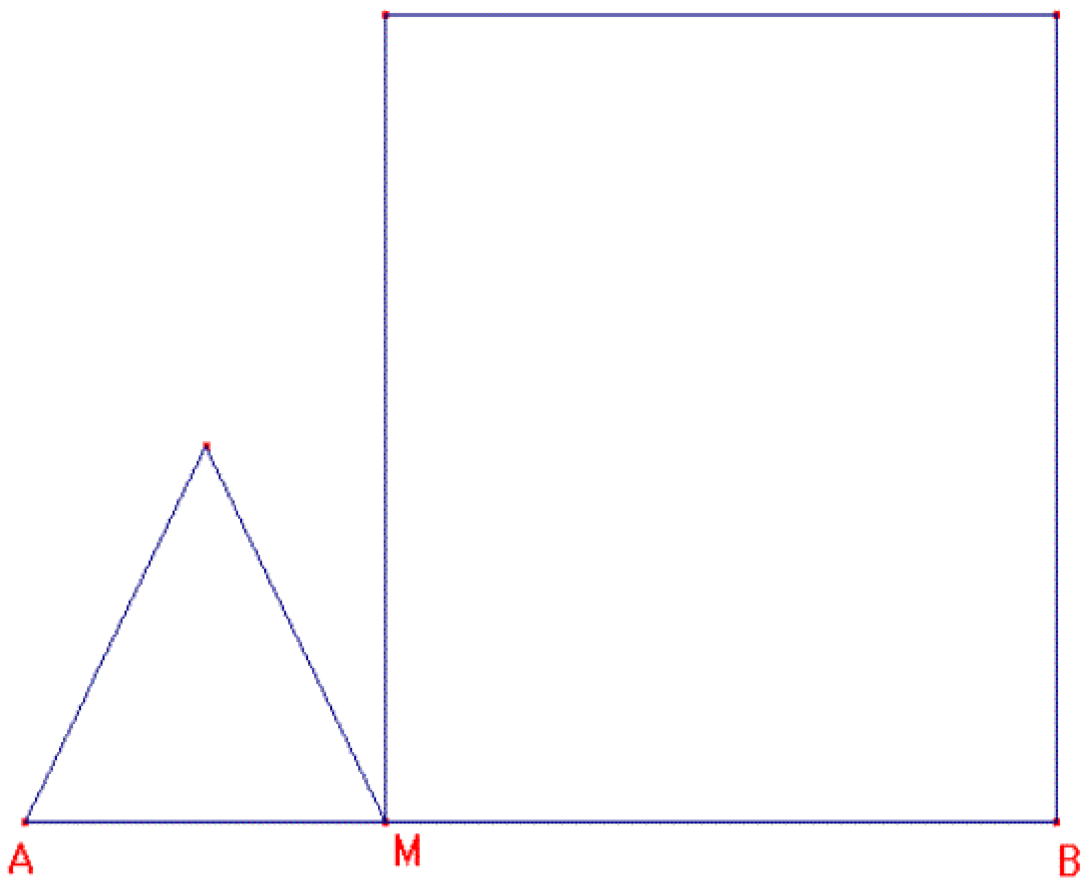
The work sessions we had about these two topics with the students were of practical work (PW), and were complementary to the mathematics normal course. We had 4 PW about the geometrical configuration of the functions, and 2 PW about the graphical configuration of such functions. In the middle of these mountings, we set up a complementary setting where we showed a geometrical configuration of some rational functions (we approached this by the simulation of the Descartes'machine). Following the scenario of Descartes'machine, we applied a test. It is important to mention that as a part of their normal course, the students had been following the traditional instruction with respect to algebraic inequation problem solving, the one consisting in the practice of a procedure which permits to establish a table of signs around the zeroes in the algebraic expressions in question and in the quotient, if such is the case. We have taken systematic observations only in the last 3 PW sessions, towards the end of the school year. These three work sessions were completely videotaped and analyzed.

The Settings

A. The interrelation settings of the algebraic representation with the geometrical configuration of functions, were of the following type:

Go to Cabri-II. Draw an equilateral triangle of side [AM], and a square of side [MB], given a segment [AB] and one point M on that segment;

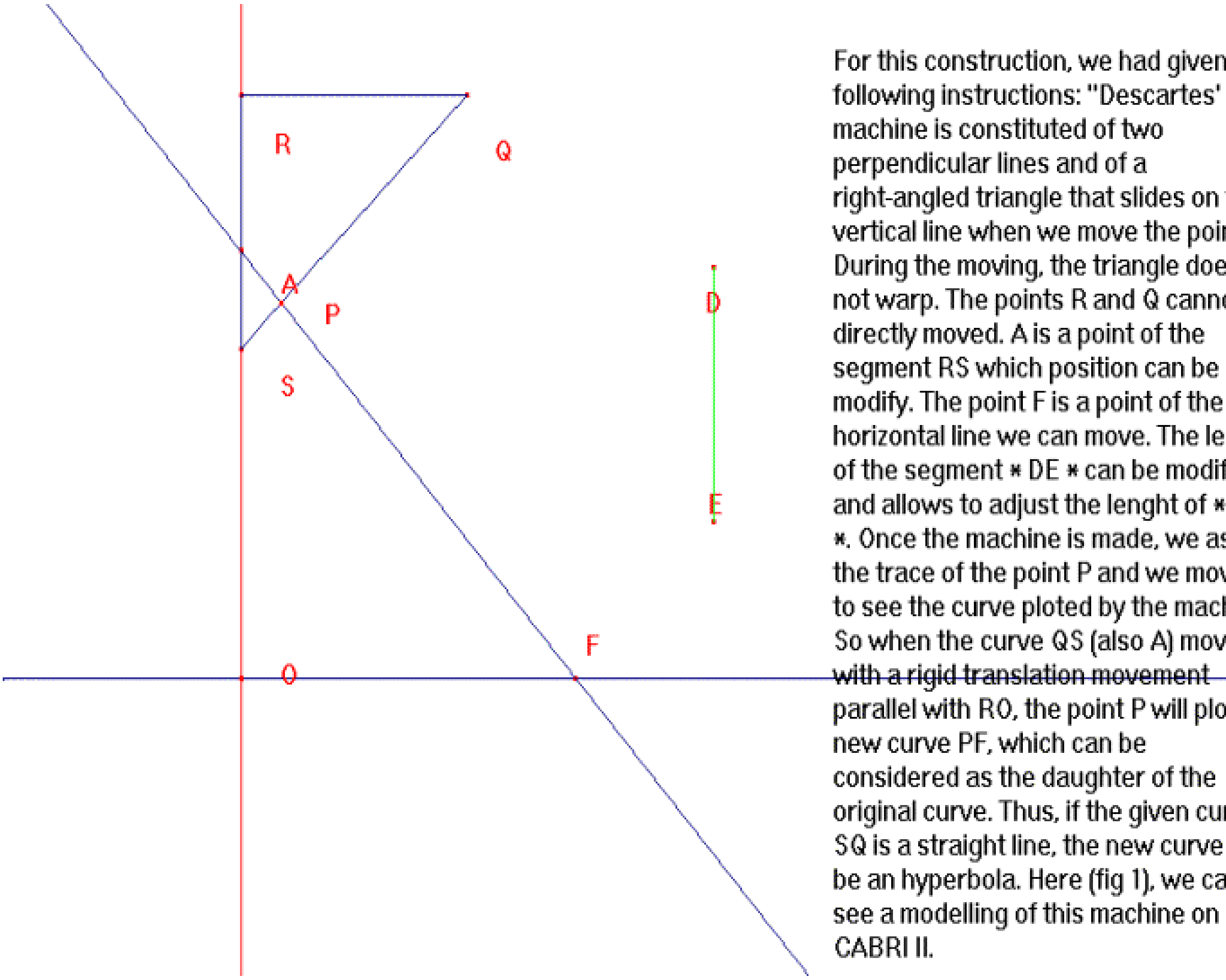
- Call longitude AM, x. What is the interval to which x belongs?
- Calculate the perimeter P1(x) and the area A1(x) of the equilateral triangle;
- Calculate the perimeter P2(x) and the area A2(x) of the square.



In fact, we hoped the students were experiencing by these settings, a kind of prototypes of functions that involved geometrical variability.

B. The interrelation settings of the algebraic representation with a graphic configuration of the functions, were of the following type:
By several of these settings, the students were experiencing prototypes of functions that involved dependence, as we might show you later.

C. A part of the complementary setting about the geometrical configuration of some rational functions (introduced by the simulation of the Descartes'machine) was the following:

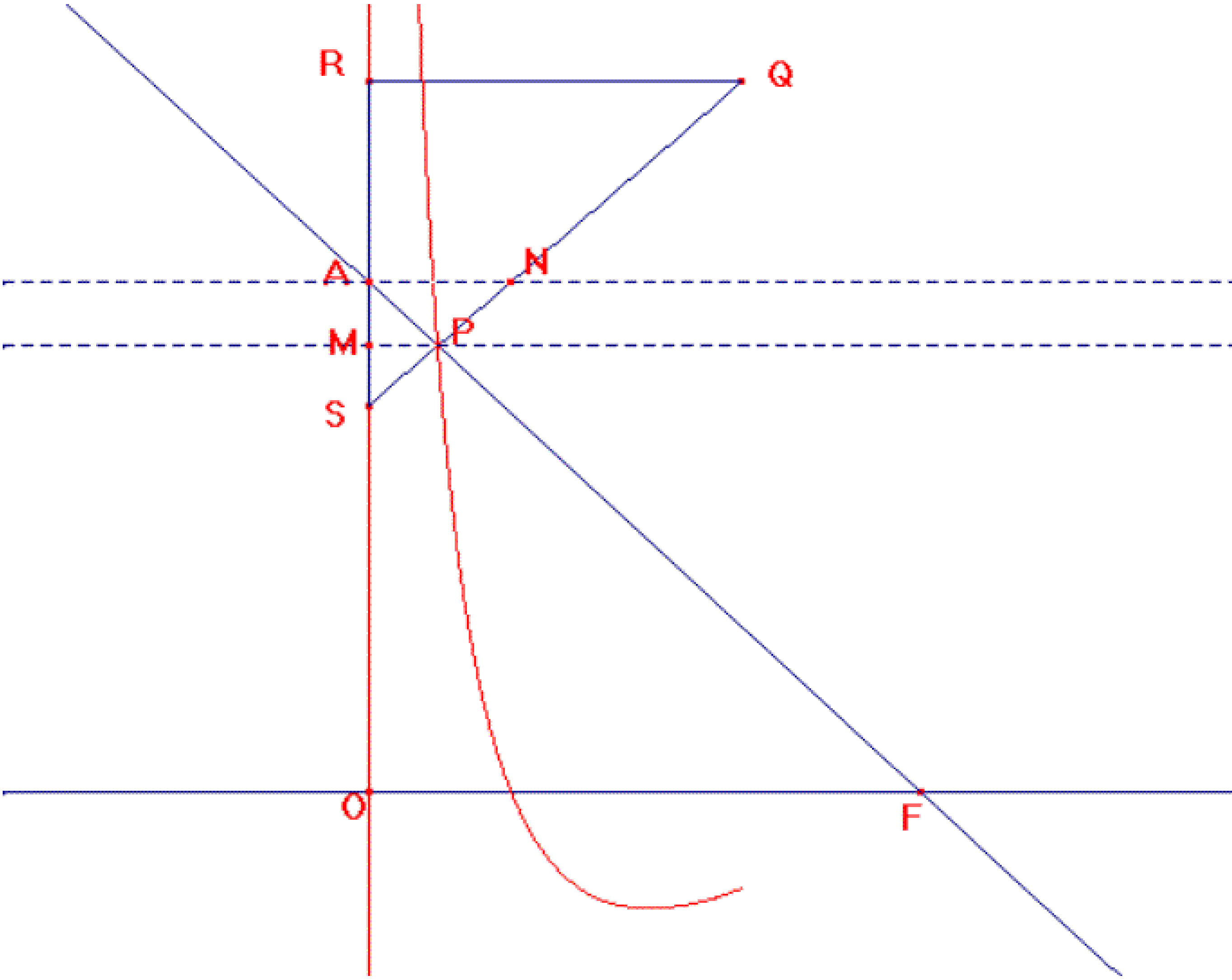


The purpose of this setting was to introduce rational functions, like 1/x-2+x = y by the means of a geometrical configuration that involved variability.

In fact, our position was that Cabri-II could be a means for the establishment of connections between graphic and algebraic representations, which were appropriate to understand the mathematical contents of the Séconde course. The establishment of connections that are necessary to carry out algebraic tasks which are usual at that school level, such as the algebraic inequation problem solving of the type $1/x < 3/x+1$ where students normally fail when approaching them merely from an algebraico-procedural perspective, which we are identifying with the procedure of setting up a table of signs.

The types of tasks through which we intended to test the establishment of connections between the graphic and the algebraic representations were the following:

(a)To obtain the equation for the hyperbole (in function of x and y) as per the method imagined by Descartes (the application of the Theorem of Thales to triangles ASN and MSP):



(b) To solve graphically equations of the type $1/x = 3/x+1$, $|5x+2| = |-2x+1|$; (c) To solve graphically inequations of the type $1/x < 3/x+1$, y $|5x+2|$ $|-2x+1|$.

What were the students doing with?

In relation to the request to obtain the equation for the hyperbole, several pupils were close to obtaining the equation of the curve. During the timed session, approximately 50-min, despite its interest and the planned guidance, the students have only been able to establish the proportionality relations asked for. It was outside the class, later, that we asked advanced pupils to finish that task. After reworking ten minutes, they wrote the equation in question: 1/x-2+x = y.

Concerning the graphical resolution of inequations within Cabri-II we can report that during the activity developed in the settings of the graphical configuration of function, we could observe the following remarkable episodes:

- In the first session, for instance, student AM, who had already placed the variable point M on the axis of the abscissas, could not mark the point corresponding to f(x) on the axis of the ordinates, a matter which at first she attributes to an anomaly in the software. In order to be able to escape this false impasse, she must have point M, the point of variable abscissa x, move. Let us now see this protocol summary between student AM and Professor P:

> AM: (after appropriately following the given instructions) But... this leads to nothing, right? Sir, this is a measuring relation!
> P: Is it, Audrai?
> AM: This does not place a point f... Why, is it that I'm wrong or not?
> P: What do you say? Yes, what is it you're saying? 13, 62? Yes.
> AM: I have indeed taken a measure relationship... The 13,62...
> P: Yes, but wait. Before you go on, according to you, where is the 13,62, if you have marked it on this axis?
> AM: Oh, but how stupid I am!
> P: Well, then, what should we do?... So, you escape, you simply escape. There, and try to see... There you are!
> Do you understand what she is doing? (The Professor is now talking to student CA, who has been working with AM for the entire session).
> CA: No.
> P: Explain to her what you have done. She doesn't understand why you've done this.
> AM: The fact is that there was a number, not the function. This is 13,62; and you mark 13,62.
> This was over there (she signals towards the top of the screen). Well, then, I move M in order to have f(x) smaller...
> P: She decreases x in order to have a smaller f(x). Do you agree? There you are.
> (Cf. Hoyos,1999).

- During the last of these two PW sequences on graphical representation, already in a situation of solving algebraic inequalities with Cabri-II, one of pupils, NS spontaneously resorted to the revision of his own resolution procedures: in the middle of the graphication activity of the algebraic expressions in question, NS decided to solve the inequation (1¸x)<(3¸(x+1)) through the usual procedure of setting up a table of signs, and to compare the results derived from the two resolution procedures applied. Ns, at this moment, had at his disposal the visualization of the graphical representation of the algebraic expressions which he had just performed in Cabri-II (see Hoyos & Capponi, 1999). To Ns's astonishment, the different procedures employed showed different results. NS demands the immediate help of the Professor, who revises what NS has done, and points out to him the errors of his decisions made only on the basis of a crossed product, followed by the construction of a table of changes of signs.

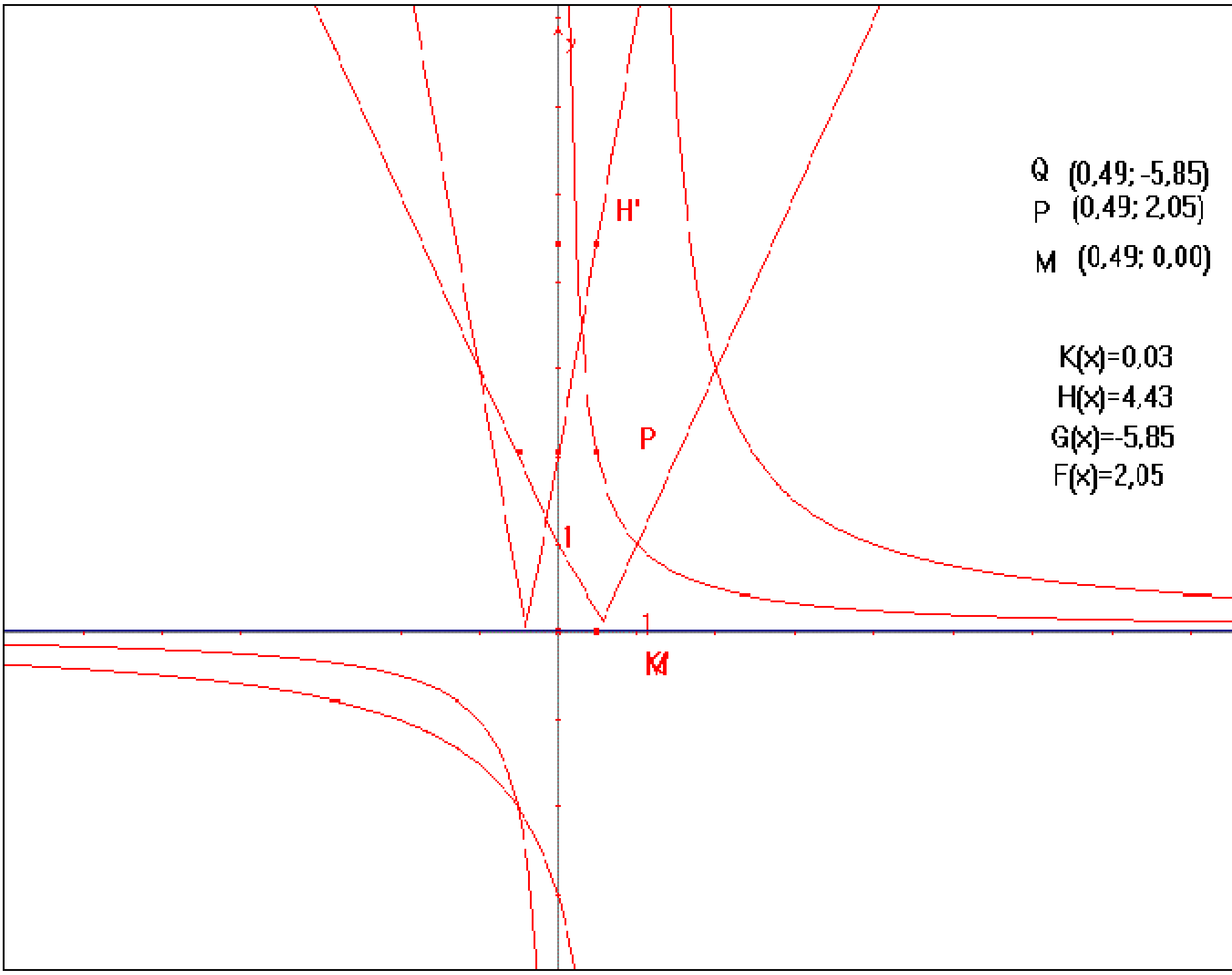
Once passed the moment of the revision of his own resolution procedures -which to us is crucial for the student's understanding--, the student was able to successfully solve by himself, within Cabri-II, the rest of the rational inequations programmed for the aforesaid session. We show below one image of the performance within Cabri-II of this pupil, NS.

Some Conclussions

We have observed then, when the pupils were working within Cabri-II the different settings we have elaborated, two actions that promoted a growth in the understanding of what functions mean:

(1) That of making point M move in a dynamic manner, to escape from an impasse or in order to visualize what is going to happen; and
(2) To make the description or to discuss that was happening within the software to the teacher or to the classmates. This was made in order to make sense out of the parallel visualization of the effect of moving M over against to the ordinate f(x) point of the y axis, in the case of the graphical configuration of the functions; and over against to the dynamic changes of the figures in the case of the geometrical configurations of the function.

Finally, the analysis of the pupils' interplay within Cabri-II software that we have tried to show here, have indicated that perhaps the connections that students establish between the different types of function representations are due, to the experience promouted when they were working within the software, because the kind of support involved by the prototypes they were performing: we are speaking of experiencing variability and dependence, which are both structural aspects to the construction of the function object (Freudenthal, 1983), or which are a part of the object perspective (Sfard, 1991; Kieran, 1992) of the concept of function.



Q (0,49; -5,85)
P (0,49; 2,05)
M (0,49; 0,00)
K(x)=0,03
H(x)=4,43
G(x)=-5,85
F(x)=2,05
H'
P
1
1
K
M